Communications

Transient Fields of Thin Cylindrical Antennas

Abstract—The time variation of the radiation field of a thin cylindrical antenna driven by a dc pulse is investigated theoretically. Computed values of the radiation field are presented graphically for successive instants of time as a function of the angle between the antenna axis and the position vector of the distant point in space and for different angles as a function of time.

I. Introduction

Previous studies of the transient behavior of cylindrical antennas have presented the time history of the radiation field of a vertical monopole for points along the ground plane [1]–[3]. In this paper the transient radiation field for directions other than the direction along the ground plane will be obtained.

II. Analysis

It will be assumed that the antenna is a cylindrical tube of infinite conductivity, length $2h$, and radius a, excited at the center by an applied potential V across a zero-width gap or by a so-called delta-function generator.

The far-zone electric field for an $\exp(j\omega t)$ time dependence for such an antenna is given by [4]

$$E_\theta(r,\theta;\omega) = VF(\theta;\omega)\exp(-jkr)/r \tag{1}$$

where

$$\begin{aligned}F(\theta;\omega) &= \tfrac{1}{2}\sin\theta[\ln(2/ka) - 0.5572 - j(\pi/2) - \ln(\sin\theta)]^{-1}\\ &\quad\cdot\left[\frac{1}{2}\left\{\frac{1-\cos(kh(1+\cos\theta))}{1+\cos\theta} + \frac{1-\cos(kh(1-\cos\theta))}{1-\cos\theta}\right\}\right.\\ &\quad\left.+ j\left(\frac{1+C}{2}\right)\left\{\frac{\sin(kh(1+\cos\theta))}{1+\cos\theta} + \frac{\sin(kh(1-\cos\theta))}{1-\cos\theta}\right\}\right].\end{aligned} \tag{2}$$

Here (r,θ) are the usual spherical coordinates, k is the wave number, and C is a constant. Derivation of $E_\theta(r,\theta;\omega)$ was based on the assumption that the vector potential on the cylindrical surface $\rho = a$ is given by

$$A(a,z) = (V/2c)[\exp(-jk|z|) + C\cos kz][u(z+h) - u(z-h)] \tag{3}$$

where c is the velocity of light and $u(z)$ is the usual unit step function.

An accurate expression for the constant C can only be obtained from a solution of the antenna integral equation. Hallén [5], [6] and Wu [4] have solved the integral equation and have obtained an expression for C. Hallén's expression for the constant C and for the antenna input admittance Y_a will be used since they are for computational purposes simpler than that of Wu's. Both expressions

$$C = -\frac{4(\Omega+\Omega_0)^{-1}}{(\Omega_0)^{-1}\exp(2kh) + 2(\Omega+\Omega_0+2\ln 2)^{-1}} \tag{4}$$

$$Y_a = \tfrac{1}{60}[(\Omega_0)^{-1} + 2C(\Omega+\Omega_0)^{-1}] \tag{5}$$

where

$$\Omega = 2\ln(2h/a), \quad \Omega_0 = 2[\ln(1/ka) - 0.5772 - j(\pi/2)] \tag{6}$$

have been approximately evaluated by assuming the antenna is thin in the sense that $ka \ll 1$.

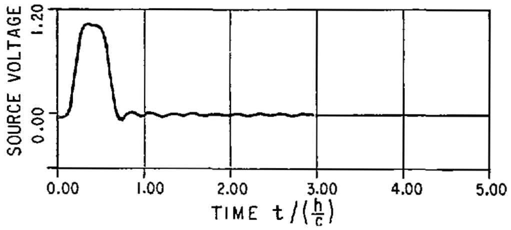

Fig. 1. Waveform represents source voltage $v_c(t)$.

The computation time to find the inverse Fourier transform of (1) can be greatly reduced if the source function is synthesized on a periodic basis in terms of a finite number of discrete frequency components. We let the dc pulse train, represented by

$$v_c(t) = \tau/T + \operatorname{Re}\sum_{n=1}^{N} V_{cn}\exp(j\omega_n t) \tag{7}$$

where

$$V_{cn} = \frac{2}{n\pi}\sin\left(\frac{n\pi\tau}{T}\right)\left[\frac{\sin(n\pi/2N)}{n\pi/2N}\right]\exp[-j(\omega_n\tau/2)] \tag{8}$$

drive a monopole of length h oriented vertically above a perfectly conducting ground plane. Then the far-zone electric field is given by

$$\begin{aligned}e_\theta(r,\theta;t) &= \operatorname{Re}\sum_{n=1}^{N} V_{cn}[0.5 + Z_cY_a(\omega_n)]^{-1}\\ &\quad\cdot F(\theta;\omega_n)\exp[j\omega_n(t - r/c)]/r.\end{aligned} \tag{9}$$

Here $\omega_n = 2\pi n/T$, T is the period, τ is the pulsewidth, and Z_c is the source impedance. Re signifies that only the real part of the summation is to be used.

III. Results

Equation (9) multiplied by r to remove the dependence of field amplitude on distance to the observation point was evaluated for $N = 100, T = 25(h/c)$, and $\tau = 0.4(h/c)$. This value for T allows zero steady-state field to be established after each pulse when $Z_c = 50$ or 300 ohms. Furthermore, these values define a pulse whose highest frequency component is $(1.2/h)$ GHz, and whose rise time is approximately $2h/3$ ns (see Fig. 1). The time t

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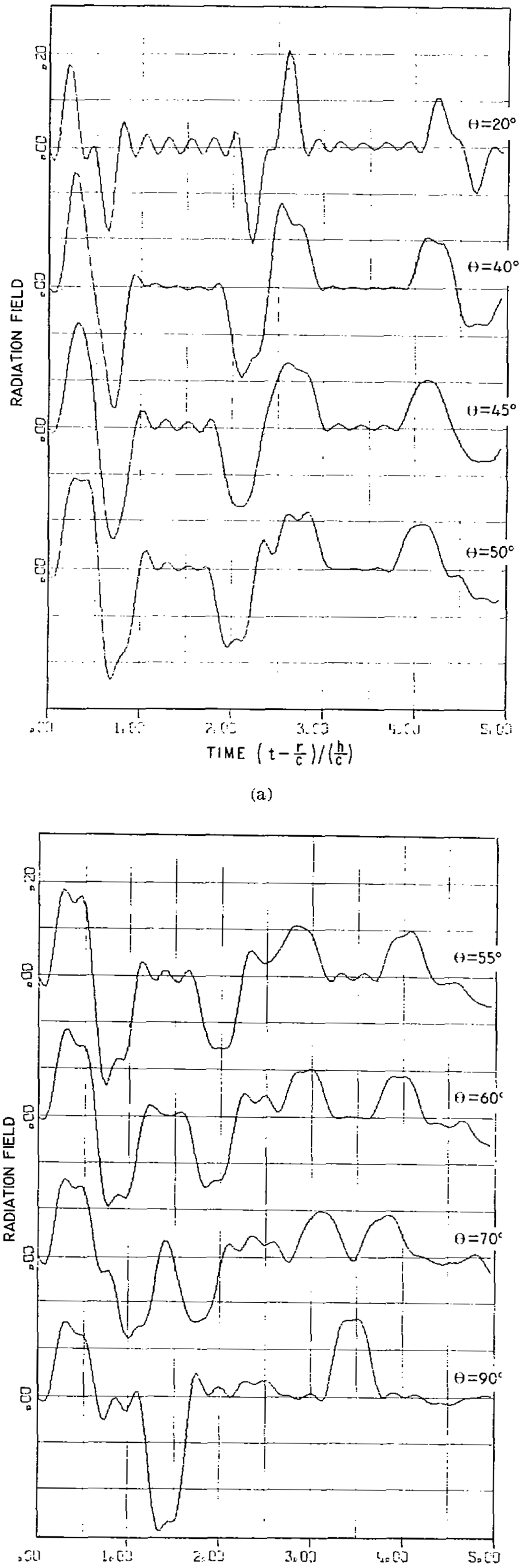

(a)

(b)

Fig. 2. Waveforms of (a) and (b) represent the time and angle θ dependence of the radiation field, $r \times e_\theta(r,\theta;t)$ volts, for the excitation $v_c(t)$ shown in Fig. 1, for $Z_c = 50$ ohms, and for $h/a = 904$.

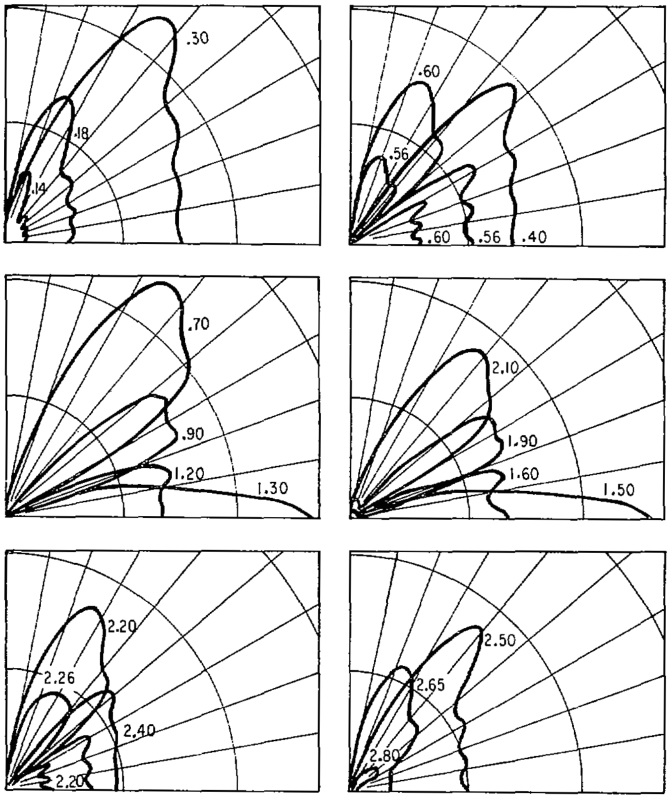

Fig. 3. Waveforms represent radiation field magnitude, $r \times |e_\theta(r,\theta;t)|$ volts, for successive instants of time, for excitation $v_c(t)$ shown in Fig. 1, for $Z_c = 300$ ohms, and for $h/a = 904$. Numbers are equal to $(t - r/c)/(h/c)$. Vertical line emanating from the lower left-hand corner coincides with antenna axis $\theta = 0°$, the horizontal line, with the ground $\theta = 90°$. Scale: 1 inch = 0.06 volts.

in (7) and (9) has been replaced by $t - 0.2\ (h/c)$, so that all of the leading edge of the pulse will be seen for $t > 0$.

Fig. 2 shows the time dependence of the radiation field at distant observation points in different θ relative to the positive axis of the antenna. Since the radiation field is proportional to the surface integral of the time derivative of the antenna current distribution, the observed time delay between the effect of the time derivative of the leading edge of the pulse and that of the trailing edge will be proportional to $\tau \cos \theta$. Thus the observed pulses on the radiation field waveform should be of pulsewidth $\tau(h/c) \cos \theta$.

The second pulse in the radiation field waveform is negative, as represented in Fig. 2; it originated when the current pulse was undergoing reflection at the end of the antenna. The third pulse in the waveform of the radiation field, which is also negative, results from the image current pulse on the other half of the antenna as it is reflected from the other end. The difference in time delay of the effect of the reflection of the pulse at one end and that at the other end is approximately $2(h/c) \cos \theta$. As the observation point approaches the direction normal to the antenna axis ($\theta = 90°$), the second and third pulses in the waveform of the radiation field approach coincidence. Later significant pulses in the waveform of the radiation field arise from subsequent reflections at the ends.

The time dependence of the magnitude of the radiation field as a function of the angle θ for successive instants of time is shown in Fig. 3. To minimize the duration of the transient field, a source with an impedance of 300 ohms was assumed.

IV. SUMMARY

The transient radiation fields of a thin cylindrical antenna have been determined for excitation by a dc pulse. The fields were presented graphically as a function of time for successive directions relative to the antenna axis, and as a function of the direction angle for successive instants of time. These results supplement the transient field waveforms obtained in previous studies, which gave the fields only for directions normal to the antenna axis.

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References

[1] H. J. Schmitt, "Transients in cylindrical antennas," *Proc. IEE* (London), Monograph 377E, pt. C, pp. 292–298, April 1960.
[2] H. J. Schmitt, C. W. Harrison, Jr., and C. S. Williams, Jr., "Calculated and experimental response of thin cylindrical antennas to pulse excitation," *IEEE Trans. Antennas and Propagation*, vol. AP-14, pp. 120–127, March 1966.
[3] C. W. Harrison and R. W. P. King, "On the transient response of infinite cylindrical antenna," *IEEE Trans. Antennas and Propagation*, vol. AP-15, pp. 301–302, March 1967.
[4] T. T. Wu, "Theory of the dipole antenna and the two-wire transmission line," *J. Math. Phys.*, vol. 2, pp. 550–574, July/August 1961.
[5] E. Hallén, *Electromagnetic Theory*. New York: Wiley, 1962, p. 465.
[6] ——, "Exact solution of the antenna equation," *Trans. Roy. Inst. Tech.*, Stockholm, no. 183, 1961.

A Circular Loop Antenna with Traveling-Wave Current Distribution

In earlier investigations on traveling-wave loop antenna [1], [2], attention was focused on a single turn circular loop with a total phase change of $2\pi n$ radians, n being an integer. In this case the radiation pattern is omnidirectional in the plane of the loop. In this communication, analysis has been carried out for the more general case of a circular loop with an arbitrary phase change for one complete revolution. The radiation pattern and gain have been computed for different loop lengths and different phase variations over the loop.

The lossless circular loop antenna, shown in Fig. 1, has been assumed to have a traveling wave current distribution of the form $I = I_0 \exp(-jH_1\phi') \exp(j\omega t)$, where H_1 is real and may assume any value. The total change in phase for one revolution around the loop is $2\pi H_1$ radians. For such a loop, the expressions for the components of the vector potential at a distant point P are given by

$$
\begin{aligned}
A_\phi &= A_0 \int_0^{2\pi} \cos(\phi - \phi') \exp(-jH_1\phi') \exp[jl \cos(\phi - \phi')]\, d\phi' \\
A_\theta &= A_0 \cos\theta \int_0^{2\pi} \sin(\phi - \phi') \exp(-jH_1\phi') \exp[jl \cos(\phi - \phi')]\, d\phi' \\
A_r &= A_0 \sin\theta \int_0^{2\pi} \sin(\phi - \phi') \exp(-jH_1\phi') \exp[jl \cos(\phi - \phi')]\, d\phi'
\end{aligned}
\tag{1}
$$

where

$$A_0 = I_0 R \exp[j(\omega t - \beta r_0)]/4\pi r_0$$

$l = H_2 \sin\theta = \beta R \sin\theta$, H_2 is the length of the loop in wavelengths, and β is the phase constant. Equation (1) for the vector potential may be employed to find the field intensities for any value of H_1 and H_2. The integrals in the above expressions lead to different forms depending upon whether H_1 is an integer or not, irrespective of the value of H_2. The two cases have been considered separately and the expressions for the field intensities derived from (1) are presented below.

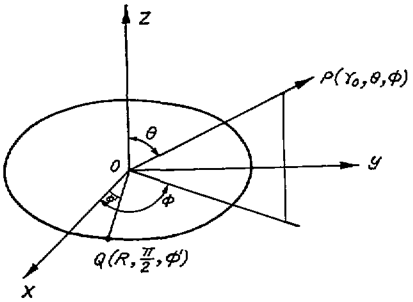

Fig. 1. Coordinate system for a circular loop antenna.

Case I

For $H_1 = m$, an integer, from (1), the components of the far-zone electric field intensity are given by [1]

$$E_\phi = -jE_0 J_m'(l), \quad E_\theta = E_0 m \cos\theta [J_m(l)/l] \tag{2}$$

where

$$E_0 = (I_0 R \omega\mu / 2r_0) \exp\{j(\omega t - \beta r_0 - m\phi + [(m-1)/2]\pi)\}$$

and $J_m(l)$ is the Bessel function of the first kind.

The results show that the radiation pattern is omnidirectional in the plane of the loop. This corresponds to the simplified modes sometimes used to represent radiating regions on spiral antennas.

Case II

For the case H_1 is not an integer, from (1), the components of the far zone electric field intensity have been found to be of the form

$$
\begin{aligned}
E_\phi &= E_1 \left[\frac{-J_1(l)}{2H_1} + \sum_{p=1}^{\infty} \frac{(j)^p J_p'(l)}{(H_1^2 - p^2)} (H_1 \cos p\phi - jp \sin p\phi) \right] \\
E_\theta &= E_1 \cos\theta \left[\sum_{p=1}^{\infty} \frac{(j)^p p}{(H_1^2 - p^2)} \frac{J_p(l)}{l} (H_1 \sin p\phi + jp \cos p\phi) \right]
\end{aligned}
\tag{3}
$$

where

$$E_1 = (I_0 R \omega\mu / 2\pi r_0) \exp[j(\omega t - \beta r_0 - \pi/2)][\exp(-j2\pi H_1) - 1].$$

The above expressions show how the far zone radiation pattern depends on H_1 and H_2. The principal plane radiation patterns have been computed for different combinations of H_1 and H_2. For loop lengths of $H_2 \leq 1.5$, the two infinite series given in (3) converge rapidly and hence can be approximated with negligible error by retaining only the terms involving the Bessel functions up to the third order. It has been determined with the help of the IBM 1620 digital computer that for loop lengths less than 0.9λ (i.e., $H_2 \leq 0.9$) as the total change in phase is increased from 0 to 1.8π radians (i.e., $0 \leq H_1 \leq 0.9$), the direction of maximum radiation shifts from $\theta = 90°$ to $\theta = 0°$ but always remains in the $\phi = 90°$ plane. The horizontal plane ($\theta = 90°$) and the vertical plane ($\phi = 90°$) (Fig. 1) radiation patterns for looplengths of 0.1λ, 0.5λ, and 0.9λ (i.e., $H_2 = 0.1$, 0.5, and 0.9, respectively) and for different values of total phase change are given in Figs. 2 and 3, respectively. When $H_1 \ll H_2$, the radiation pattern is almost the same as that of a loop with uniform current. It is seen from Fig. 2 that for a given loop, as the total phase is increased from zero, the horizontal plane radiation pattern ceases to be omnidirectional and for a particular value of H_1, the radiation pattern is most directive and as H_1 is further increased and approaches unity,

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